The thermalization process of an atom with the thermal radiation field

2007 J. Phys. A: Math. Theor. 40 13217

(http://iopscience.iop.org/1751-8121/40/44/003)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **40** (2007) 13217–13230 doi:10.1088/1751-8113/40/44/003

The thermalization process of an atom with the thermal radiation field

G Flores-Hidalgo

Instituto de Ciências Exatas, Universidade Federal de Itajubá, Av BPS 1303, 37500-903 Itajubá-MG, Brazil

E-mail: gfloreshidalgo@unifei.edu.br

Received 19 July 2007, in final form 19 September 2007
Published 16 October 2007
Online at stacks.iop.org/JPhysA/40/13217

Abstract
We study the time evolution of an atom suddenly coupled to a thermal radiation field. As a simplified model of the atom-electromagnetic field system we use a system composed of a harmonic oscillator linearly coupled to a scalar field in the framework of the recently introduced dressed coordinates and dressed states. We show that the time evolution of the thermal expectation values for the occupation number operators depends exclusively on the probabilities associated with the emission and absorption of field quanta. In particular, the time evolution of the number operator associated with the atom is given in terms of the probability of remaining in the first excited state and the decay probabilities from this state by emission of field quanta of frequencies ω_k. Also, it is shown that independent of the initial state of the atom, it thermalizes with the thermal radiation field in a time scale of the order of the inverse coupling constant.

PACS numbers: 03.65.Yz, 05.70.Ln, 05.30.Jp

1. Introduction

The study of systems out of thermal equilibrium has been since a long-time ago one of the main active areas in physics. The actual interest ranging from the condensed matter physics to cosmology. In most cases the interest is in the thermalization process, the determination of the relevant time scales involved, together with an understanding of the generation of entropy and particle production in non-equilibrium dissipative systems interacting with an environment. However, despite the importance associate with these processes, non-equilibrium problems are still poorly understood [1]. The nontrivial non-equilibrium dynamics of fields, for instance, have diverse applications, finding use, e.g., in the studies concerning the recent experiments in the ultra-relativistic heavy-ion collision [2]; applications to the current problems of parametric resonance and particle production in cosmology [3]; or in the context of the recent studies involving the intrinsic dissipative nature of interacting fields [4–6]. In addition to that, typical problems we have in mind to study are those related to the nontrivial out-of-thermal equilibrium dynamics associated with phase transitions in different physical systems. As a few examples, we may cite include the current applications to the study of formation of Bose–Einstein condensates after a temperature quench [7], or in the study of the dynamics of coupled fields displaced from their ground states as determined by their free energy densities [8]. For recent attempts to solve some related problems to the study of systems out of thermal equilibrium (see [9–14, 16–18]), where use has been made of either analytical or numerical approaches in the context of specific or general models. For example, numerical studies have been performed in specific field theoretical models in [9–12], where the problems of equilibration and thermalization have been studied. On the other hand, in [14] the role of chaos as a mechanism for quantum thermalization has been considered. By supposing the validity of Berry's conjecture [15] it has been shown that a gas of rarefied hard-spheres approaches a Maxwell–Boltzmann, Bose–Einstein or Fermi–Dirac distribution according to whether the wavefunctions are taken to be non-symmetric, completely symmetric or completely antisymmetric functions of the particle position.

In recent works, in analogy with the renormalized fields in quantum field theory, the concepts of dressed coordinates and dressed states have been introduced [19–21]. These concepts have been introduced in the context of an atom, approximated by an harmonic oscillator, linearly coupled to a scalar field, the whole system being confined in a spherical cavity of the diameter L. In terms of dressed coordinates, dressed states have been defined as the physically measurable states. The dressed states having the physical correct property of stability of the oscillator (atom) ground state in the absence of field quanta (the quantum vacuum). For a recent clear explanation see [25]. Also, the formalism has proved the technical advantage of allowing an exact computation of the probabilities associated with the different oscillator (atom) radiation processes [26]. For example, we obtained easily the probability of the atom to decay spontaneously from the first excited state to the ground state for arbitrary coupling constant, weak or strong and for arbitrary cavity size. For weak coupling constant and in the continuum limit $L \to \infty$ we obtained the old-known result: $\mathrm{e}^{-\Gamma t}$ [19]. Also, considering a cavity of sufficiently small radius [20], the method accounted for, the experimentally observed, inhibition of the spontaneous decaying processes of the atom [22, 23]. In [24, 25], the concept of dressed coordinates and states has been extended to the case in which nonlinear interactions between the oscillator and the field modes are taken into account. Furthermore, in [27] we considered the oscillator electromagnetic field interaction model and in [28] dressed coordinates and states have been introduced in the path integral formalism.

The aim of the present work is to study the thermalization process in the framework of the aforementioned dressed coordinates and states. The physical situation that we have in mind is an atom (approximated by an harmonic oscillator) initially in an arbitrary state, suddenly coupled to a thermal radiation field (approximated by an infinite set of harmonic oscillators at thermal equilibrium). Then, the purpose is to study the time evolution of this initial state. Fundamental questions that we have to solve are: is the atom reaches a final equilibrium state? and if this is the case, what is the meaning of this final equilibrium state? In relation with these questions, it is also important to know the time necessary for the atom to thermalize with the thermal radiation field. By solving exactly this model we expect to gain some insight to solve more complicated problems.

As already stated, in this paper we will treat a specific model for an atom coupled to a thermal radiation field. Initially, the system is described by a density operator of the form

$$\hat{\rho} = \hat{\rho}_0 \otimes \hat{\rho}_\beta, \tag{1}$$

where $\hat{\rho}_0$ is the density operator for the atom, which can be in an arbitrary pure or mixed state and $\hat{\rho}_\beta$ is the density operator for the radiation field at thermal equilibrium at some given temperature β^{-1}. We specify below the form of $\hat{\rho}_\beta$. At some time, that we take as $t = 0$, the atom is suddenly coupled to the thermal radiation field, afterward (the density operator of) the total system evolves according to the Liouville–Von Neumann equation. An equivalent description is to maintain constant the density operator and take the operators (related to the physical observables) as time dependent. Then, these operators evolve in time according to the Heisenberg equation of motion

$$\frac{\partial}{\partial t}\hat{O}(t) = \mathrm{i}[\hat{H}, \hat{O}(t)], \tag{2}$$

where $\hat{O}(t)$ is a time-dependent operator associated with some physical observable and $\hat{H}$ is the Hamiltonian for the atom-electromagnetic field system. As a model for this system we consider the one with Hamiltonian given by

$$\hat{H} = \frac{1}{2}\left(\hat{p}_0^2 + \omega_0^2\hat{q}_0^2\right) + \frac{1}{2}\sum_{k=1}^{N}\left(\hat{p}_k^2 + \omega_k^2\hat{q}_k^2 - 2c_k\hat{q}_k\hat{q}_0\right) + \frac{1}{2}\sum_{k=1}^{N}\frac{c_k^2}{\omega_k^2}\hat{q}_0^2, \tag{3}$$

where the limit $N \to \infty$ is understood, the subscript 0 refers to the atom approximated by an harmonic oscillator of frequency ω_0 and $k = 1, 2, \ldots, N$ refer to the harmonic field modes. Also, we take $\omega_k = 2\pi k/L$, $c_k = \eta\omega_k$, $\eta = \sqrt{2g\Delta\omega}$ and $\Delta\omega = \omega_{k+1} - \omega_k = 2\pi/L$, where g is a frequency dimensional coupling constant. At the end we will take the continuum limit $L \to \infty$. The last term in equation (3) assures the positiveness of the Hamiltonian and it can be seen as a frequency renormalization of the harmonic oscillator [29, 30].

A similar model to the one given by equation (3) has been used repeatedly from time to time as a simplified model to describe the quantum Brownian motion [31–34], the decoherence problem and other related problems [35, 36]. However, in all these previous works no use has been made of the dressed coordinates. As we explain in the following section, when considering the Hamiltonian given by equation (3) as the one for an atom–field electromagnetic field system, the introduction of dressed (renormalized) coordinates will be necessary in order to guarantee the stability of the atom ground state in the absence of field quanta.

Along this paper we use natural units $\hbar = c = k_B = 1$.

2. Dressed (renormalized) coordinates and the dressed density operator

To make this paper self-contained in this section we define what has been called dressed coordinates and dressed states in [19–21]. To understand the necessity of introducing dressed coordinates in the system atom-electromagnetic field system described by Hamiltonian (3), take $c_k = 0$. In this case, the resulting free Hamiltonian admits the following eigenfunctions:

$$\begin{aligned}\psi_{n_0 n_1\cdots n_N}(q) &\equiv \langle q|n_0, n_1, \ldots, n_N\rangle \\ &= \prod_{\mu=0}^{N}\left[\left(\frac{\omega_\mu}{\pi}\right)^{1/4}\sqrt{\frac{2^{-n_\mu}}{n_\mu!}}H_{n_\mu}(\sqrt{\omega_\mu}q_\mu)\,\mathrm{e}^{-\frac{1}{2}\omega_\mu q_\mu^2}\right].\end{aligned} \tag{4}$$

The physical meaning of $\psi_{n_0 n_1\cdots n_N}(q)$ in this case is clear, it represents the atom in its n_0th excited level and n_k photons of frequencies ω_k. Now, consider the state $\psi_{n_0 0\cdots 0}(q)$: the excited atom in the quantum vacuum. We know from experience that any excited level of the atom is unstable. The explanation of this fact is that the atom is not isolated from interacting with the quantum electromagnetic field. This interaction in our model is given by the linear coupling of q_0 with q_k. Obviously, when we take into account this interaction any state of the type

$\psi_{n_0 0\cdots 0}(q)$ is rendered unstable. But there is a problem, the state $\psi_{00\cdots 0}(q)$, that represents the atom in its ground state and no photons, is also unstable contradicting the experimental fact of the stability of the atom ground state. What is wrong? The first thing that comes to our mind is to think that the model given by equation (3) is wrong. Certainly, we know that the correct theory to describe this physical system is quantum electrodynamics. On the other hand, such a description could be extremely complicated. If we aim to maintain the model as simple as possible and still insist in describing it by the Hamiltonian given in equation (3), what we can do in order to take into account the stability of the atom ground state? The answer lies in the spirit of the renormalization program in quantum field theory: the coordinates q_μ that appear in the Hamiltonian are not the physical ones, they are bare coordinates. We introduce dressed (or renormalized) coordinates, q_0' and q_k', respectively for the dressed atom and the dressed photons. We define these coordinates as the physically meaningful ones. In terms of these coordinates we define the dressed states by

$$\begin{aligned}\psi_{n_0 n_1\cdots n_N}(q') &\equiv \langle q'|n_0, n_1, \ldots, n_N\rangle_d \\ &= \prod_{\mu=0}^{N}\left[\left(\frac{\omega_\mu}{\pi}\right)^{1/4}\sqrt{\frac{2^{-n_\mu}}{n_\mu!}}H_{n_\mu}(\sqrt{\omega_\mu}q_\mu')\,\mathrm{e}^{-\frac{1}{2}\omega_\mu(q_\mu')^2}\right],\end{aligned} \tag{5}$$

where the subscript d means dressed state. The dressed states given by equation (5) are defined as the physically measurable states and describe in general, the physical atom in the n_0th excited level and n_k physical photons of frequencies ω_k. Obviously, in the limit in which the coupling constant c_k vanishes the renormalized coordinates q_μ' must approach the bare coordinates q_μ. Now, in order to relate the bare and dressed coordinates we have to use the physical requirement of stability of the dressed ground state. The dressed ground state will be stable if it is defined as eigenfunction of the interacting Hamiltonian given by equation (3). Also the dressed ground state must be the one of minimum energy, that is, it must be defined as being identical (or proportional) to the ground-state eigenfunction of the interacting Hamiltonian. From this definition, one can construct the dressed coordinates in terms of the bare ones. Then, the first step in order to obtain the dressed coordinates is to solve for the ground-state eigenfunction of the Hamiltonian given in equation (3). This bilinear Hamiltonian can be diagonalized by introducing normal coordinates and momenta $\hat{Q}_r$ and $\hat{P}_r$,

$$\hat{q}_\mu = \sum_{r=0}^{N} t_\mu^r \hat{Q}_r, \qquad \hat{p}_\mu = \sum_{r=0}^{N} t_\mu^r \hat{P}_r, \qquad \mu = (0, k), \tag{6}$$

where $\{t_\mu^r\}$ is an orthonormal matrix whose elements are given by [37]

$$t_k^r = \frac{c_k}{(\omega_k^2 - \Omega_r^2)} t_0^r, \qquad t_0^r = \left[1 + \sum_{k=1}^{N}\frac{c_k^2}{(\omega_k^2 - \Omega_r^2)^2}\right]^{-\frac{1}{2}} \tag{7}$$

with Ω_r being the normal frequencies corresponding to the collective modes of the coupled system and given as solutions of the equation

$$\omega_0^2 - \Omega_r^2 = \sum_{k=1}^{N}\frac{c_k^2\Omega_r^2}{\omega_k^2(\omega_k^2 - \Omega_r^2)}. \tag{8}$$

In terms of normal coordinates and momenta the Hamiltonian given by equation (3) reads as

$$\hat{H} = \frac{1}{2}\sum_{r=0}^{N}(\hat{P}_r^2 + \Omega_r^2\hat{Q}_r^2), \tag{9}$$

then, the eigenfunctions of the Hamiltonian are given by

$$\phi_{n_0 n_1 \cdots n_N}(Q) \equiv \langle Q | n_0, n_1, \ldots, n_N \rangle_c = \prod_{r=0}^{N} \left[\left(\frac{\Omega_r}{\pi} \right)^{1/4} \sqrt{\frac{2^{-n_r}}{n_r!}} H_{n_r}(\sqrt{\Omega_r} Q_r)\, \mathrm{e}^{-\frac{1}{2}\Omega_r Q_r^2} \right], \tag{10}$$

where the subscript c means collective state. Now, using the definition of the dressed coordinates: $\psi_{00\cdots 0}(q') \propto \phi_{00\ldots 0}(Q)$ and using equations (5) and (10), we get $\mathrm{e}^{-\frac{1}{2}\sum_{\mu=0}^{N} \omega_\mu (q'_\mu)^2} = \mathrm{e}^{-\frac{1}{2}\sum_{r=0}^{N} \Omega_r Q_r^2}$, from which the dressed coordinates are obtained as

$$q'_\mu = \sum_{r=0}^{N} \sqrt{\frac{\Omega_r}{\omega_\mu}} t_\mu^r Q_r. \tag{11}$$

2.1. The dressed density operator

If no use is made of the dressed coordinates and states, the density operator for the radiation field at thermal equilibrium in equation (1) would be given by

$$\hat{\rho}_\beta = Z_\beta^{-1} \exp\left[-\beta \sum_{k=1}^{N} \omega_k \left(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2} \right) \right], \tag{12}$$

where $\hat{a}_k$ and $\hat{a}_k^\dagger$ are annihilation and creation operators and given by

$$\hat{a}_\mu = \frac{1}{\sqrt{2\omega_\mu}} \hat{p}_\mu - \mathrm{i}\sqrt{\frac{\omega_\mu}{2}} \hat{q}_\mu \tag{13}$$

$$\hat{a}_\mu^\dagger = \frac{1}{\sqrt{2\omega_\mu}} \hat{p}_\mu + \mathrm{i}\sqrt{\frac{\omega_\mu}{2}} \hat{q}_\mu. \tag{14}$$

In equation (12) $Z_\beta = \prod_{k=1}^{N} z_\beta^k$ is the partition function of the thermal radiation field, where

$$z_\beta^k = \mathrm{Tr}_k[\mathrm{e}^{-\beta\omega_k(\hat{a}_k^\dagger \hat{a}_k + 1/2)}] = \frac{1}{2\sinh\left(\frac{\beta_k \omega_k}{2}\right)}. \tag{15}$$

Also, the density operator $\hat{\rho}_0$ for the atom would be written in terms of the coordinates q_0.

However, as explained above, in the context of an atom-electromagnetic field system and described by Hamiltonian given by equation (3) it is necessary to redefine what the physical coordinates are for the atom and field modes. Then, instead of the density operator given by equation (12), we have to consider the one written in terms of dressed coordinates q'_k, as the physically density operator for the radiation field at thermal equilibrium,

$$\hat{\rho}_\beta = Z_\beta^{-1} \exp\left[-\beta \sum_{k=1}^{N} \omega_k \left(\hat{a}_k'^\dagger \hat{a}'_k + \frac{1}{2}. \right) \right], \tag{16}$$

where $\hat{a}'_k$ and $\hat{a}_k'^\dagger$ are dressed annihilation and creation operators and given in terms of the dressed coordinates q'_k by

$$\hat{a}'_\mu = \frac{1}{\sqrt{2\omega_\mu}} \hat{p}'_\mu - \mathrm{i}\sqrt{\frac{\omega_\mu}{2}} \hat{q}'_\mu \tag{17}$$

$$\hat{a}_\mu'^\dagger = \frac{1}{\sqrt{2\omega_\mu}} \hat{p}'_\mu + \mathrm{i}\sqrt{\frac{\omega_\mu}{2}} \hat{q}'_\mu, \tag{18}$$

where in position representation $\hat{p}'_\mu = -\mathrm{i}\frac{\partial}{\partial q'_\mu}$. Also, the density operator for the atom must be taken as the one written in terms of the dressed coordinate q'_0.

Now, we are ready to study the time evolution of thermal expectation values for relevant physical operators. We will be mainly interested in the present work in the study of the time evolution of the thermal expectation value of the time-dependent number occupation operator associated with the dressed oscillator (the atom) $\hat{a}'^{\dagger}_0(t)\hat{a}'_0(t)$.

3. The thermalization process

We consider as the initial state of the atom-electromagnetic system that given by equation (1), where $\hat{\rho}_0$ describes an arbitrary, pure or mixed, state for the atom and $\hat{\rho}_\beta$, given by equation (16), describes the thermal radiation field at some given temperature β^{-1}. With this condition we can state the thermalization problem for this system as follows: (i) the initial state given by equation (1) will evolve in time to a final equilibrium state? and (ii) if the system evolves to a final equilibrium state, is this an state of thermal equilibrium?. Also we would like to know the mean-time necessary for the system to reach a final thermal equilibrium state. To answer these questions we have to solve for the time dependence of the density operator or alternatively solve the Heisenberg equation to obtain the time dependence of relevant operators.

Since any operator can be written in terms of annihilation and creation operators, it will be sufficient to solve for the time-dependent annihilation and creation operators in order to solve the out of thermal equilibrium problem. Using the Heisenberg equation of motion, equation (2), we have for the time-dependent annihilation operator $\hat{a}'_\mu(t)$,

$$\frac{\partial}{\partial t}\hat{a}'_\mu(t) = \mathrm{i}[\hat{H}, \hat{a}'_\mu(t)] \tag{19}$$

and a similar equation for $\hat{a}'^{\dagger}_\mu(t)$. Obviously at $t = 0$, $\hat{a}'_\mu(0)$ is given by equation (17). This equation can be written, using equations (6) and (11), as

$$\hat{a}'(0) = \sum_{r,\nu=0}^{N}\left(\frac{t^r_\mu t^r_\nu}{\sqrt{2\Omega_r}}\hat{p}_\nu - \mathrm{i}\sqrt{\frac{\Omega_r}{2}}t^r_\mu t^r_\nu\hat{q}_\nu\right). \tag{20}$$

In order to solve equation (19) we write $\hat{a}'_\mu(t)$ as

$$\hat{a}'_\mu(t) = \sum_{\nu=0}^{N}(B(t)_{\mu\nu}\hat{p}_\nu + \dot{B}_{\mu\nu}(t)\hat{q}_\nu), \tag{21}$$

where $B(t)_{\mu\nu}$ is a time-dependent c-number and the dot means derivative with respect to time. Replacing equations (3) and (21) in equation (19), working with the commutators and identifying identical operators in both sides of the resultant equation, we obtain the following coupled equations for $B_{\mu\nu}(t)$:

$$\ddot{B}_{\mu0}(t) + \left(\omega_0^2 + \sum_{k=1}^{N}\frac{c_k^2}{\omega_k^2}\right)B_{\mu0}(t) - \sum_{k=1}^{N}c_k B_{\mu k}(t) = 0 \tag{22}$$

and

$$\ddot{B}_{\mu k}(t) + \omega_k^2 B_{\mu k}(t) - c_k B_{\mu0}(t) = 0. \tag{23}$$

Note that above equations are identical to the classical equations of motion for the bare coordinates q_μ that can be obtained using the Hamilton equations of motion for the Hamiltonian given by equation (3). Then we can decouple equations (22) and (23) with the same matrix $\{t_\mu^r\}$ that diagonalizes the Hamiltonian (3), that is, we can write for $B_{\mu\nu}(t)$,

$$B_{\mu\nu}(t) = \sum_{r=0}^{N} t_\mu^r C_\nu^r(t) \tag{24}$$

and replacing the above equation in equations (22) and (23), these equations decouple into

$$\ddot{C}_\mu^r(t) + \Omega_r^2 C_\mu^r(t) = 0, \tag{25}$$

from which we obtain $C_\mu^r(t) = a_\mu^r\,\mathrm{e}^{\mathrm{i}\Omega_r t} + b_\mu^r\,\mathrm{e}^{-\mathrm{i}\Omega_r t}$. Then, substituting this expression into equation (24) we obtain

$$B_{\mu\nu}(t) = \sum_{r=0}^{N} t_\nu^r \left(a_\mu^r\,\mathrm{e}^{\mathrm{i}\Omega_r t} + b_\mu^r\,\mathrm{e}^{-\mathrm{i}\Omega_r t}\right). \tag{26}$$

The time-independent coefficients a_μ^r and b_μ^r are determined by the initial conditions at $t = 0$ for $B_{\mu\nu}(t)$ and $\dot{B}_{\mu\nu}(t)$. From equations (20) and (21) we find that these initial conditions are

$$B_{\mu\nu}(0) = \sum_{r=0}^{N} \frac{t_\mu^r t_\nu^r}{\sqrt{2\Omega_r}}, \tag{27}$$

$$\dot{B}_{\mu\nu}(0) = -\mathrm{i}\sum_{r=0}^{N} \sqrt{\frac{\Omega_r}{2}}\, t_\mu^r t_\nu^r. \tag{28}$$

Using the above initial conditions in equation (26) and the orthonormality property of the matrix $\{t_\mu^r\}$ we obtain $a_\mu^r = 0$ and $b_\mu^r = \frac{t_\mu^r}{\sqrt{2\Omega_r}}$. Replacing these values in equation (26) we get

$$B_{\mu\nu}(t) = \sum_{r=0}^{N} \frac{t_\mu^r t_\nu^r}{\sqrt{2\Omega_r}}\,\mathrm{e}^{-\mathrm{i}\Omega_r t}. \tag{29}$$

Using equation (29) in equation (21) we can get easily

$$\hat{a}'_\mu(t) = \sum_{\nu=0}^{N} f_{\mu\nu}(t)\hat{a}'_\nu, \tag{30}$$

where

$$f_{\mu\nu}(t) = \sum_{r=0}^{N} t_\mu^r t_\nu^r\,\mathrm{e}^{-\mathrm{i}\Omega_r t}. \tag{31}$$

Now, we can compute the time evolution of the expectation value corresponding to the dressed occupation number operator $\hat{n}_\mu(t) = \hat{a}'^{\dagger}_\mu(t)\hat{a}'_\mu(t)$,

$$n_\mu(t) = \mathrm{Tr}\left[\hat{a}'^{\dagger}_\mu(t)\hat{a}'_\mu(t)\hat{\rho}_0 \otimes \hat{\rho}_\beta\right], \tag{32}$$

where $\hat{\rho}_0$ is the dressed density operator corresponding to the atom and $\hat{\rho}_\beta$ is the dressed density operator for the thermal radiation field and given by equation (16). To compute the trace in equation (32) we choose the basis $|n_0, n_1, \ldots, n_N\rangle_d$. From equation (30) and its Hermitian conjugate we have

$$\begin{aligned}\hat{a}'^{\dagger}_\mu(t)\hat{a}'_\mu(t) &= \sum_{\nu,\rho=0}^{N} f^*_{\mu\rho}(t) f_{\mu\nu}(t)\hat{a}'^{\dagger}_\rho\hat{a}'_\nu \\ &= \sum_{\nu=0}^{N} |f_{\mu\nu}(t)|^2\hat{a}'^{\dagger}_\nu\hat{a}'_\nu + \sum_{\nu\neq\rho} f^*_{\mu\rho}(t) f_{\mu\nu}(t)\hat{a}'^{\dagger}_\nu\hat{a}'_\rho.\end{aligned} \tag{33}$$

In the basis $|n_0, n_1, \ldots, n_N\rangle_d$, the second term in the above equation gives no contribution for equation (32). Then, replacing equation (33) in equation (32) we obtain easily

$$n_\mu(t) = |f_{\mu 0}(t)|^2 n_0(0) + \sum_{k=1}^{N} |f_{\mu k}(t)|^2 n_k(0), \tag{34}$$

where the initial distributions for the dressed atom and field modes are given respectively by

$$\begin{aligned} n_0(0) &= \mathrm{Tr}_0\left(\hat{a}_0'^{\dagger}\hat{a}_0'\hat{\rho}_0\right) \\ &= \sum_{n=0}^{\infty} n_d\langle n|\hat{\rho}_0|n\rangle_d \end{aligned} \tag{35}$$

and

$$n_k(0) = \frac{\mathrm{Tr}_k\left(\hat{a}_k'^{\dagger}\hat{a}_k'\,\mathrm{e}^{-\beta\omega_k(\hat{a}_k'^{\dagger}\hat{a}_k'+1/2)}\right)}{\mathrm{Tr}_k\left(\mathrm{e}^{-\beta\omega_k(\hat{a}_k'^{\dagger}\hat{a}_k'+1/2)}\right)} = \frac{1}{\mathrm{e}^{\beta\omega_k} - 1}. \tag{36}$$

Setting $\mu = 0$ in equation (34), we obtain for the time-dependent thermal expectation value of the occupation number operator, corresponding to the atom,

$$n_0(t) = |f_{00}(t)|^2 n_0(0) + \sum_{k=1}^{N} |f_{0k}(t)|^2 n_k(0). \tag{37}$$

In early references it has been shown that $|f_{00}(t)|^2$ is the probability of the atom to remain at time t in the first excited level, whereas $|f_{0k}|^2$ is the probability decay of the atom from the first excited level to the ground state by emission of a field quanta of frequency ω_k [19–21]. Then, equation (37) suggests a clear physical interpretation in terms of these probabilities. Also equation (34) can be interpreted in the same way.

For the frequency field modes given in the paragraph after equation (3) and in the continuum limit $L \to \infty$ the coefficients $f_{00}(t)$ and $f_{0k}(t)$ are calculated in the appendix. We obtain the following values (equations (A.17) and (A.19)):

$$f_{00}(t) = \left(1 - \frac{\mathrm{i}\pi g}{2\kappa}\right)\mathrm{e}^{-\mathrm{i}\kappa t - \pi g t/2} + 2\mathrm{i}g J(t) \tag{38}$$

and

$$f_{0k}(t) = \sqrt{2g\Delta\omega}\omega_k\left[\frac{\left(1 - \frac{\mathrm{i}\pi g}{2\kappa}\right)\mathrm{e}^{-\mathrm{i}\kappa t - \pi g t/2}}{\left[\omega_k^2 - \left(\kappa - \frac{\mathrm{i}\pi g}{2}\right)^2\right]} - \frac{\mathrm{e}^{-\mathrm{i}\omega_k t}}{\left[\omega_k^2 - \omega_0^2 + \mathrm{i}\pi g\omega_k\right]}\right] + 2\mathrm{i}g\sqrt{2g\Delta\omega}\omega_k I(\omega_k, t), \tag{39}$$

where $\kappa = \sqrt{\omega_0^2 - \frac{\pi^2}{4}g^2}$,

$$J(t) = \int_0^\infty \mathrm{d}y \frac{y^2\,\mathrm{e}^{-yt}}{\left(y^2 + \omega_0^2\right)^2 - \pi^2 g^2 y^2} \tag{40}$$

and

$$I(\omega_k, t) = \int_0^\infty \mathrm{d}y \frac{y^2\,\mathrm{e}^{-yt}}{\left[\left(y^2 + \omega_0^2\right)^2 - \pi^2 g^2 y^2\right]\left(y^2 + \omega_k^2\right)}. \tag{41}$$

Replacing equations (38) and (39) in equation (34) we obtain in the continuum limit $\Delta\omega \to 0$, $N \to \infty$,

$$n_0(t) = P_{00}(t) n_0(0) + \int_0^\infty \mathrm{d}\omega \frac{P_{0\omega}(t)}{(\mathrm{e}^{\beta\omega} - 1)}, \tag{42}$$

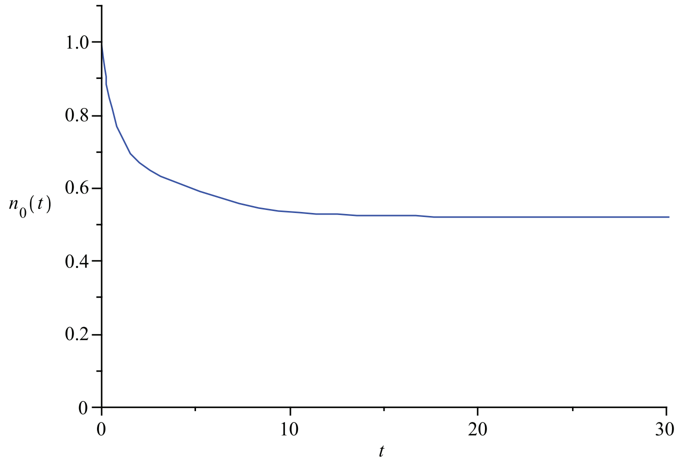

Figure 1. Time behavior for $n_0(t)$ given by equation (42), for $n_0 = 1$, $\omega_0 = \beta = 1$ and $g = 0.1$

where

$$P_{00}(t) = \frac{\omega_0^2}{\kappa^2}\,\mathrm{e}^{-\pi g t} - 2gJ(t)\,\mathrm{e}^{-\pi g t/2}\left[2\sin(\kappa t) + \frac{\pi g}{\kappa}\cos(\kappa t)\right] + 4g^2J^2(t), \tag{43}$$

$$\begin{aligned} P_{0\omega}(t) = 2\frac{g}{\kappa}\omega^2 \Bigg\{ & \frac{\kappa^2 + \omega_0^2\,\mathrm{e}^{-\pi g t}}{\kappa K(\omega)} - \frac{\mathrm{e}^{-\pi g t/2}}{K^2(\omega)}\Big(\big[2\kappa\big(\omega^2 - \omega_0^2\big)^2 + \pi^2 g^2\omega\big(\omega^2 + \omega_0^2\big)\big]\cos[(\omega-\kappa)t] \\ & + \pi g\big(\omega^2 - \omega_0^2\big)\big(\omega^2 + \omega_0^2 - 2\kappa\omega\big)\sin[(\omega-\kappa)t] + 2gI(\omega,t)K(\omega) \\ & \times\big[2\kappa\big(\omega^2 - \omega_0^2\big)\sin(\kappa t) + \pi g\big(\omega^2 + \omega_0^2\big)\cos(\kappa t)\big]\Big) \\ & + 4\,g\kappa\frac{I(\omega,t)}{K(\omega)}\big[\big(\omega^2 - \omega_0^2\big)\sin(\omega t) + \pi g\omega\cos(\omega t)\big] + 4\kappa g^2 I^2(\omega,t)\Bigg\}, \end{aligned} \tag{44}$$

and

$$K(\omega) = \big(\omega^2 - \omega_0^2\big)^2 + \pi^2 g^2\omega^2. \tag{45}$$

Although it is not possible to compute analytically the integral in equation (42) we can perform numerical calculations, for example in figure 1, we display the time behavior for $n_0(t)$ for $n_0(0) = 1$, $\omega_0 = \beta = 1$ and $g = 0.1$. Note that for large t, $n_0(t)$ approaches a fixed value. This behavior is general as one can see by taking the limit $t \to \infty$ in equation (42), where it is obtained a well-defined limit. This means that the atom reaches a final equilibrium state. Also, in this limit the term $P_{00}(t)$ proportional to $n_0(0)$ vanishes, that is, the final equilibrium distribution is independent of the initial atom density operator, $\hat{\rho}_0$, it depends exclusively on the thermal field degrees of freedom. As shown in [19–21] $P_{00}(t)$ goes to zero almost exponentially in a time of the order π/g. Taking $t \to \infty$ in equation (42) we get

$$n_0(\infty) = 2g\int_0^\infty \mathrm{d}\omega \frac{\omega^2}{[(\omega^2 - \omega_0)^2 + \pi^2 g^2\omega^2](\mathrm{e}^{\beta\omega} - 1)}. \tag{46}$$

Now, the question is about the physical meaning of the equilibrium value given by equation (46). To answer this question we compute the thermal expectation value of the number operator $\hat{a}_0'^{\dagger}\hat{a}_0'$, in the case in which the atom-electromagnetic field system is at thermal equilibrium at some given temperature θ^{-1}. In this case, the density operator is given by

$$\hat{\rho}_\theta = \frac{\mathrm{e}^{-\theta\hat{H}}}{\mathrm{Tr}(\mathrm{e}^{-\theta\hat{H}})}, \tag{47}$$

where $\hat{H}$ is given by equation (3). We want to compute

$$n_0 = \frac{\mathrm{Tr}\left(\hat{a}_0'^{\dagger}\hat{a}_0'\,\mathrm{e}^{-\theta\hat{H}}\right)}{\mathrm{Tr}(\mathrm{e}^{-\theta\hat{H}})}. \tag{48}$$

To compute the above expression we write $\hat{H}$ as

$$\hat{H} = \sum_{r=0}^{N}\left(\hat{A}_r^{\dagger}\hat{A}_r + \frac{1}{2}\right)\Omega_r, \tag{49}$$

where $\hat{A}_r$ and $\hat{A}_r^{\dagger}$ are the normal annihilation and creation operators and given by

$$\hat{A}_r = \frac{1}{\sqrt{2\Omega_r}}\hat{P}_r - \mathrm{i}\sqrt{\frac{\Omega_r}{2}}\hat{Q}_r \tag{50}$$

$$\hat{A}_r^{\dagger} = \frac{1}{\sqrt{2\Omega_r}}\hat{P}_r + \mathrm{i}\sqrt{\frac{\Omega_r}{2}}\hat{Q}_r. \tag{51}$$

Now, using equation (11) and from equations (17), (18) and (50), (51) we find that

$$\hat{a}_\mu' = \sum_{r=0}^{N} t_\mu^r \hat{A}_r, \qquad \hat{a}_\mu'^{\dagger} = \sum_{r=0}^{N} t_\mu^r \hat{A}_r^{\dagger}. \tag{52}$$

Using above expressions in equation (48) and computing the trace by using the basis $|n_0, n_1, \ldots, n_N\rangle_c$, which are eigenvectors of $\hat{H}$, we find easily

$$n_0 = \sum_{r=0}^{N} \frac{\left(t_0^r\right)^2}{\mathrm{e}^{\theta\Omega_r} - 1} \tag{53}$$

and in the continuum limit we get (see the appendix, equation (A.20))

$$n_0 = 2g\int_0^\infty \mathrm{d}x \frac{x^2}{[(x^2-\omega_0)^2 + \pi^2 g^2 x^2](\mathrm{e}^{\theta x} - 1)}. \tag{54}$$

In the case in which $\theta = \beta$, equations (54) and (46) are identical. Then, we conclude from above calculations that the atom reaches a final thermal equilibrium distribution, it thermalizes with the thermal radiation field at temperature β^{-1}. Note that for weak coupling $g \ll \omega_0$, we can obtain from equation (46) or (54),

$$n(\infty) \approx \frac{1}{\mathrm{e}^{\beta\omega_0} - 1}, \tag{55}$$

a Bose–Einstein distribution, an expected textbook result.

4. Conclusions

In this work, we have shown that an atom (approximated by the dressed harmonic oscillator) initially in any arbitrary state and suddenly coupled to a thermal radiation field evolves in time to a final thermal equilibrium state. The mean time, necessary for this to occur, can be roughly estimated from equations (43)–(44) and is of the order π/g, an intuitively expected result. Also, we have found a physically suggestive result for the time evolution of the thermal expectation value of the dressed occupation number operators, equations (34) and (37). In general, this time evolution is given in terms of the time-dependent probabilities associated with the emission and absorption of field quanta.

We would like to remark that if one works in bare coordinates a similar result to the one given by equation (34) is obtained but with different time-dependent coefficients and non-homogeneous terms, i.e., there are time-dependent terms that do not multiply $n_\mu(0)$ (see, for example, equation (117) of [37]). However there are two problems with this result. First, $n_0(t)$ is discontinuous at $t = 0$, see equations (117) and (B6–B7) of [37]. Second, one of the non-homogeneous terms is divergent (see, for example, equation (121) of the cited reference and comments below this equation). We can solve this ultraviolet divergence by a renormalization procedure, that is equivalent to normal ordering the annihilation and creation operators. The discontinuity problem of $n_0(t)$ can be attributed to the sudden coupling of the bare atom with the field modes. However in the dressed coordinate approach, as one can note from equation (42), $n_0(t)$ is continuous at $t = 0$, even so also in this case the atom suddenly couples to the thermal bath. Then, the discontinuity of $n_0(t)$ can be viewed as a pathology of the bare coordinates approach. Also $n_0(t)$, in the dressed coordinates formalism is finite, as one can see from equation (42) or from figure 1, and no further renormalization is required. Finally, in bare coordinates, the time-dependent coefficients that appear in the expression for $n_0(t)$ are not related to physically significant quantities. All these advantages suggest us that the dressed coordinate approach is physically more adequate than the bare coordinates approach.

Acknowledgments

The author acknowledges Y W Milla for the assistance to perform the numerical computation displayed in figure 1. This work is supported by FAPEMIG (Fundação de Amparo à Pesquisa do Estado de Minas Gerais).

Appendix. The continuum limit

We want to compute, in the continuum limit, the sums of the type

$$R_{\mu\nu} = \sum_{r=0}^{N} t_\mu^r t_\nu^r \mathcal{R}_{\mu\nu}(\Omega_r), \tag{A.1}$$

where $\mathcal{R}_{\mu\nu}(\Omega)$ is an analytic function of Ω. For this end we define a function $W(z)$,

$$W(z) = z^2 - \omega_0^2 + \sum_{k}^{N} \frac{\eta^2 z^2}{\omega_k^2 - z^2}. \tag{A.2}$$

From equations (7) and (8) we can note that the Ω_r's are the roots of $w(z)$. For complex values of z and using $\eta^2 = 2g\Delta\omega$, we can write equation (A.2) in the continuum limit as

$$W(z) = z^2 - \omega_0^2 + 2gz^2 \int_0^\infty \frac{\mathrm{d}\omega}{\omega^2 - z^2}. \tag{A.3}$$

For complex values of z the above integral is well defined and can be evaluated easily using the Cauchy theorem, obtaining

$$W(z) = \begin{cases} z^2 + \mathrm{i}g\pi z - \omega_0^2, & \mathrm{Im}(z) > 0 \\ z^2 - \mathrm{i}g\pi z - \omega_0^2, & \mathrm{Im}(z) < 0. \end{cases} \tag{A.4}$$

We start by computing $R_{00}(t)$,

$$R_{00} = \sum_{r=0}^{N} \left(t_0^r\right)^2 \mathcal{R}_{00}(\Omega_r). \tag{A.5}$$

From the expression for t_0^r, given in (7) and equation (A.2) it is easy to show that

$$\left(t_0^r\right)^2 = \frac{2\Omega_r}{W'(\Omega_r)}, \tag{A.6}$$

where the prime means derivative with respect to the argument. Since the Ω_r's are the roots of $W(z)$, we can write equation (A.5) as

$$R_{00} = \frac{1}{\mathrm{i}\pi} \oint_C \mathrm{d}z \frac{z\mathcal{R}_{00}(z)}{W(z)}, \tag{A.7}$$

where C is a counterclockwise contour in the z-plane that encircles the real positive roots Ω_r, that is, a contour that encircles the real positive axis. The integral in equation (A.7) can be evaluated choosing a contour that lies just below and above of the real positive axis. Below the real positive axis we have $z = \alpha - \mathrm{i}\epsilon$ and above $z = \alpha + \mathrm{i}\epsilon$, where α is real positive and $\epsilon \to 0^+$. Then, we have for equation (A.7),

$$R_{00} = \frac{1}{\mathrm{i}\pi} \int_0^\infty \mathrm{d}\alpha \left[\frac{(\alpha - \mathrm{i}\epsilon)\mathcal{R}_{00}(\alpha - \mathrm{i}\epsilon)}{W(\alpha - \mathrm{i}\epsilon)} - \frac{(\alpha + \mathrm{i}\epsilon)\mathcal{R}_{00}(\alpha + \mathrm{i}\epsilon)}{W(\alpha + \mathrm{i}\epsilon)} \right]. \tag{A.8}$$

From equation (A.4) we get for $W(\alpha - \mathrm{i}\epsilon)$ and $W(\alpha - \mathrm{i}\epsilon)$ respectively in the limit $\epsilon \to 0^+$,

$$\begin{aligned} W(\alpha + \mathrm{i}\epsilon) &= \alpha^2 - \omega_0^2 + \mathrm{i}g\pi\alpha, \\ W(\alpha - \mathrm{i}\epsilon) &= \alpha^2 - \omega_0^2 - \mathrm{i}g\pi\alpha. \end{aligned} \tag{A.9}$$

Taking the limit $\epsilon \to 0^+$ in equation (A.8) and using equation (A.9) we get

$$R_{00} = 2g \int_0^\infty \mathrm{d}\alpha \frac{\alpha^2 \mathcal{R}_{00}(\alpha)}{\left(\alpha^2 - \omega_0^2\right)^2 + g^2\pi^2\alpha^2}. \tag{A.10}$$

As a check that equation (A.10) is correct we take the case $\mathcal{R}_{00} = 1$ and using Cauchy theorem it is easy to show that the above integral is 1, as expected from the orthonormality property of the matrix $\left\{t_\mu^r\right\}$.

Next we compute $R_{0k}(t)$,

$$R_{0k} = \sum_{r=0}^{N} t_0^r t_k^r \mathcal{R}_{0k}(\Omega_r). \tag{A.11}$$

Using the expressions for t_0^r and t_k^r, as given by (7), in equation (A.11) we obtain

$$\begin{aligned} R_{0k} &= \eta\omega_k \sum_{r=0}^{N} \frac{\left(t_0^r\right)^2 \mathcal{R}_{0k}(\Omega_r)}{\left(\omega_k^2 - \Omega_r^2\right)} \\ &= \frac{\eta\omega_k}{\mathrm{i}\pi} \oint_C \mathrm{d}z \frac{z\mathcal{R}_{0k}(z)}{\left(\omega_k^2 - z^2\right) W(z)}, \end{aligned} \tag{A.12}$$

where in the second line the pole at $z = \omega_k$ gives a zero contribution since $W(\omega_k)$ as given by equation (A.2) or (A.3) is infinity. Evaluating equation (A.12) by choosing the same contour as in the evaluation of $R_{00}(t)$, we get

$$R_{0k} = -\frac{\eta\omega_k}{\mathrm{i}\pi} \int_0^\infty \mathrm{d}\alpha \left[\frac{(\alpha - \mathrm{i}\epsilon)\mathcal{R}_{0k}(\alpha - \mathrm{i}\epsilon)}{W(\alpha - \mathrm{i}\epsilon)\left[(\alpha - \mathrm{i}\epsilon)^2 - \omega_k^2\right]} - \frac{(\alpha + \mathrm{i}\epsilon)\mathcal{R}_{0k}(\alpha + \mathrm{i}\epsilon)}{W(\alpha + \mathrm{i}\epsilon)\left[(\alpha + \mathrm{i}\epsilon)^2 - \omega_k^2\right]} \right]. \tag{A.13}$$

Using equation (A.9) in equation (A.13), we obtain

$$R_{0k} = -\frac{\eta\omega_k}{\mathrm{i}\pi}\int_0^\infty \mathrm{d}\alpha \left[\frac{\alpha\mathcal{R}_{0k}(\alpha)}{\left(\alpha - \frac{\mathrm{i}\pi g}{2} - \kappa\right)\left(\alpha - \frac{\mathrm{i}\pi g}{2} + \kappa\right)(\alpha - \mathrm{i}\epsilon - \omega_k)(\alpha - \mathrm{i}\epsilon + \omega_k)} - \frac{\alpha\mathcal{R}_{0k}(\alpha)}{\left(\alpha + \frac{\mathrm{i}\pi g}{2} - \kappa\right)\left(\alpha + \frac{\mathrm{i}\pi g}{2} + \kappa\right)(\alpha + \mathrm{i}\epsilon - \omega_k)(\alpha + \mathrm{i}\epsilon + \omega_k)}\right], \tag{A.14}$$

where $\kappa = \sqrt{\omega_0^2 - \frac{\pi^2}{4}g^2}$. To check the validity of equation (A.14) we take $\mathcal{R}_{0k} = 1$ and using Cauchy theorem it can be proved that the integral vanishes as expected from the orthonormality of the matrix $\{t_\mu^r\}$.

Now, it is straightforward to compute the coefficients $f_{\mu\nu}(t)$,

$$f_{\mu\nu}(t) = \sum_{r=0}^{N} t_\mu^r t_\nu^r \,\mathrm{e}^{-\mathrm{i}t\Omega_r}, \tag{A.15}$$

in the continuum limit. Taking $\mu = \nu = 0$ in equation (A.15) and using equation (A.10) we get

$$f_{00}(t) = 2g\int_0^\infty \mathrm{d}x \frac{x^2\,\mathrm{e}^{-\mathrm{i}tx}}{\left(x^2 - \omega_0^2\right)^2 + g^2\pi^2x^2}, \tag{A.16}$$

from which we find

$$f_{00}(t) = \left(1 - \frac{\mathrm{i}\pi g}{2\kappa}\right)\mathrm{e}^{-\mathrm{i}\kappa t - \pi g t/2} + 2\mathrm{i}g\int_0^\infty \mathrm{d}y \frac{y^2\,\mathrm{e}^{-yt}}{\left(y^2 + \omega_0^2\right)^2 - \pi^2g^2y^2}, \qquad (\kappa^2 > 0). \tag{A.17}$$

Taking $\mu = 0$, $\nu = k$ in equation (A.15) and using equation (A.14) we get

$$f_{0k} = -\frac{\eta\omega_k}{\mathrm{i}\pi}\int_0^\infty \mathrm{d}x \left[\frac{x\,\mathrm{e}^{-\mathrm{i}tx}}{\left(x - \frac{\mathrm{i}\pi g}{2} - \kappa\right)\left(x - \frac{\mathrm{i}\pi g}{2} + \kappa\right)(x - \mathrm{i}\epsilon - \omega_k)(x - \mathrm{i}\epsilon + \omega_k)} - \frac{x\,\mathrm{e}^{-\mathrm{i}tx}}{\left(x + \frac{\mathrm{i}\pi g}{2} - \kappa\right)\left(x + \frac{\mathrm{i}\pi g}{2} + \kappa\right)(x + \mathrm{i}\epsilon - \omega_k)(x + \mathrm{i}\epsilon + \omega_k)}\right]. \tag{A.18}$$

We can integrate equation (A.18) in the complex plane by using Cauchy theorem. We choice as the closed contour of integration, the path that goes in the real axis from 0 to ∞, then go to the negative imaginary axis along the part of the circle with radius $R \to \infty$ and argument $-\pi/2 < \theta < 0$ and closes the contour along the imaginary axis from $-\mathrm{i}\infty$ to the origin. Note that inside the contour of integration only the second term in the bracket of equation (A.14) has two poles at $-\mathrm{i}g\pi/2 + \kappa$ and $-\mathrm{i}\epsilon + \omega_k$. Then, we get for equation (A.18)

$$\begin{aligned} f_{0k}(t) &= -\frac{\eta\omega_k}{\mathrm{i}\pi}\left\{(-2\mathrm{i}\pi)\left[\frac{\left(\frac{\mathrm{i}\pi g}{2\kappa} - 1\right)\mathrm{e}^{-\mathrm{i}\kappa t - \pi g t/2}}{2\left[\left(\kappa - \frac{\mathrm{i}\pi g}{2}\right)^2 - \omega_k^2\right]} - \frac{\mathrm{e}^{-\mathrm{i}\omega_k t}}{2\left[\omega_k^2 + \mathrm{i}\pi g\omega_k - \omega_0^2\right]}\right]\right. \\ &\quad \left. - \int_{-\infty}^{0}\mathrm{d}y\left[\frac{y\,\mathrm{e}^{yt}}{\left(-y^2 + \pi g y - \omega_0^2\right)\left(y^2 + \omega_k^2\right)} - \frac{y\,\mathrm{e}^{yt}}{\left(-y^2 - \pi g y - \omega_0^2\right)\left(y^2 + \omega_k^2\right)}\right]\right\} \\ &= \eta\omega_k\left[\frac{\left(1 - \frac{\mathrm{i}\pi g}{2\kappa}\right)\mathrm{e}^{-\mathrm{i}\kappa t - \pi g t/2}}{\left[\omega_k^2 - \left(\kappa - \frac{\mathrm{i}\pi g}{2}\right)^2\right]} - \frac{\mathrm{e}^{-\mathrm{i}\omega_k t}}{\left[\omega_k^2 - \omega_0^2 + \mathrm{i}\pi g\omega_k\right]}\right] \\ &\quad + 2\mathrm{i}g\eta\omega_k\int_0^\infty \mathrm{d}y \frac{y^2\,\mathrm{e}^{-yt}}{\left[\left(y^2 + \omega_0^2\right)^2 - \pi^2g^2y^2\right]\left(y^2 + \omega_k^2\right)}, \ (\kappa^2 > 0). \end{aligned} \tag{A.19}$$

To compute

$$n_0 = \sum_{r=0}^{N} \frac{\left(t_0^r\right)^2}{e^{\theta\Omega_r} - 1} \tag{A.20}$$

in the continuum limit we use equation (A.10), obtaining

$$n_0 = 2g \int_0^\infty dx \frac{x^2}{[(x^2 - \omega_0)^2 + \pi^2 g^2 x^2](e^{\theta x} - 1)}. \tag{A.21}$$

References

[1] Langer J S 1992 *Solids Far from Equilibrium* ed C Godrèche (Cambridge: Cambridge University Press)
Evans D J and Morriss G P 1990 *Statistical Mechanics of Non-Equilibrium Liquids* (Edinburgh: Scottish Academic Press)
[2] Rajagopal K and Wilczek F 1995 *Nucl. Phys.* B **399** 395
Rajagopal K and Wilczek F 1993 *Nucl. Phys.* B **404** 577
[3] Kofman L, Linde A and Starobinsky A A 1997 *Phys. Rev.* D **56** 3258
[4] Gleiser M and Ramos R O 1994 *Phys. Rev.* D **50** 2441
[5] Berera A, Gleiser M and Ramos R O 1998 *Phys. Rev.* D **58** 123508
[6] Berera A and Ramos R O 2001 *Phys. Rev.* D **63** 103509
[7] Barci D G, Fraga E S and Ramos R O 2000 *Phys. Rev. Lett.* **85** 479
Barci D G, Fraga E S and Ramos R O 2002 *Laser Phys.* **12** 43
Barci D G, Fraga E S, Gleiser M and Ramos R O 2003 *Physical* A **317** 535
[8] Ramos R O and Navarro F A R 2000 *Phys. Rev.* D **62** 085016
[9] Salle M, Smit J and Vink J C 2001 *Phys. Rev.* D **64** 025016
[10] Parisi G 1997 *Europhys. Lett.* **40** 357
[11] Aarts G, Bonini G F and Wetterich Ch 2000 *Nucl. Phys.* B **587** 403
[12] Aarts G, Bonini G F and Wetterich Ch 2001 *Phys. Rev.* D **63** 025012
[13] Mosko M and Cambel V 1994 *Phys. Rev.* B **50** 8864
[14] Srednicki M 1994 *Phys. Rev.* E **50** 888
[15] Berry M V 1977 *J. Phys. A: Math. Gen.* **10** 2083
[16] Gemmer J, Otte A and Mahler G 2001 *Phys. Rev. Lett.* **86** 1927
[17] Tasaki H 1998 *Phys. Rev. Lett.* **80** 1373
[18] Scarani V, Ziman M, Stelmachovic P, Gisin N and Buzek V 2002 *Phys. Rev. Lett.* **88** 097905
[19] Andion N P, Malbouisson A P C and Neto A Mattos 2001 *J. Phys. A: Math. Gen.* **34** 3735
[20] Flores-Hidalgo G, Malbouisson A P C and Milla Y W 2002 *Phys. Rev.* A **65** 063414 (*Preprint* physics/0111042)
[21] Flores-Hidalgo G and Malbouisson A P C 2002 *Phys. Rev.* A **66** 042118 (*Preprint* quant-ph/0205042)
[22] Hulet R G, Hilfer E S and Kleppner D 1985 *Phys. Rev. Lett.* **55** 2137
[23] Jhe W, Anderson A, Hinds E A, Meschede D, Moi L and Haroche S 1987 *Phys. Rev. Lett.* **58** 666
[24] Flores-Hidalgo G and Malbouisson A P C 2003 *Phys. Lett.* A **311** 82 (*Preprint* physics/0211123)
[25] Flores-Hidalgo G and Milla Y W 2005 *J. Phys. A: Math. Gen.* **38** 7527 (*Preprint* physics/0410238)
[26] Casana R, Flores-Hidalgo G and Pimentel B M 2005 *Phys. Lett.* A **337** 1 (*Preprint* physics/0410063)
[27] Flores-Hidalgo G and Malbouisson A P C 2005 *Phys. Lett.* A **337** 37 (*Preprint* physics/0312003)
[28] Casana R, Flores-Hidalgo G and Pimentel B M 2007 *Physica* A **374** 600 (*Preprint* physics/0506223)
[29] Thirring W and Schwabl F 1964 *Ergeb. Exakt. Naturw.* **36** 219
[30] Weiss U 1993 *Quantum Dissipative Systems* (Singapore: World Scientific)
[31] Feynman R P and Vernon F L 1963 *Ann. Phys., NY* **24** 118
Feynman R P and Vernon F L 2000 *Ann. Phys., NY* **281** 547
[32] Ullersma P 1966 *Physical* **32** 27
[33] Caldeira A O and Leggett A J 1983 *Physical* A **121** 587
[34] Unruh W G and Zurek W H 1989 *Phys. Rev.* D **40** 1071
[35] Zurek W H 1991 *Phys. Today* **44** 36
[36] Hu B L, Paz J P and Zhang Y 1992 *Phys. Rev.* D **45** 2843
[37] Flores-Hidalgo G and Ramos R O 2003 *Physical* A **326** 159 (*Preprint* hep-th/0206022)